

# Top and Higgs Masses in Dynamical Symmetry Breaking<sup>1</sup>

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## ABSTRACT

A model for composite electroweak bosons is re-examined to establish approximate ranges for the initial predictions of the top and Higgs masses. Higher order corrections to this 4-fermion theory at a high mass scale where the theory is matched to the Standard Model have little effect, as do wide variations in this scale. However, including all one loop evolution and defining the masses self-consistently, at their respective poles, moves the top mass upward by some 10 GeV to near 175 GeV and the Higgs mass down by a similar amount to near 125 GeV.

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## 1. Introduction

In this paper we refine predictions for the top and Higgs masses made in an earlier work on dynamical symmetry breaking [1]. The specific 4-fermion model of dynamical symmetry breaking presented in Reference 1 (see the Lagrangian in Eq.1 below) may perhaps be ultimately viewed as the low mass limit of a gauge theory at some very high scale  $\Lambda$ , with primordial boson masses,  $M_B \sim O(\Lambda)$ . This scale then acts as an effective cutoff for the 4-fermion theory. Certainly, no explanation is presented here for the number and character of elementary fermions in the modeling nor for the large disparity in mass scales, i.e.  $m_f \ll M_B$ . Rather, a central point of our calculation is that new, composite, bosons with masses near  $2m_f$  arise naturally in the theory. These are just fermion–antifermion bound states produced by the 4-fermion interaction. This phenomenon is well described in the papers of Nambu and Jona–Lasinio on the four fermion theories [2], and has been exploited by many authors [3,4,5,6,7,8]. Since the scale  $\Lambda$  at which any new physics enters is so high, the theory is in fact a weak coupling, albeit constrained, version of the Standard Model for scales well below  $\Lambda$ .

Previously [1] we abstracted simple, asymptotic mass relationships from the 4-fermion theory, and used these as boundary conditions on the standard model renormalisation group (RG) equations. This was done at a matching scale  $\mu \sim M_{GUTS}$ , where the electroweak (EW) sector can still be treated as approximately independent of QCD ( $SU(3)_c$ ). Values for the top and Higgs masses then followed from downward evolution of the top–Higgs and Higgs-self couplings to scales near  $m_W$ , assuming no intervening structure.

In the present work we show that modifications in these asymptotic relationships, due to higher order corrections in the 4-fermion theory at the upper scale  $\mu$ , have a considerably diminished effect on  $m_t$  and  $m_H$  at their much lower scale. Also, large, several orders of magnitude, changes in  $\mu$  affect the top hardly at all and the Higgs only slightly. However, a more consistent handling of the RG evolution moves the prediction for  $m_t$  from approximately 165 GeV to nearer 175 GeV, while that for  $m_H$  moves from 140 GeV to about 125 GeV.

## 2. The 4-Fermion Theory

In Ref.1, we indicated that a 4-fermion interaction including vector terms led to rather low, well-determined masses for the top-quark and Higgs. The model is defined by the

Lagrangian:

$$\begin{aligned}\mathcal{L} = & \bar{\psi}i(\gamma \cdot \partial)\psi - \frac{1}{2}[(\bar{\psi}G_S\psi)^2 - (\bar{\psi}G_S\tau\gamma_5\psi)^2] \\ & - \frac{1}{2}G_B^2(\bar{\psi}\gamma_\mu Y\psi)^2 - \frac{1}{2}G_W^2(\bar{\psi}\gamma_\mu\tau P_L\psi)^2\end{aligned}\quad (1)$$

in which very specific vector interactions have been added to the usual scalar and pseudoscalar terms of NJL. The field operator is  $\psi = \{f_i\}$ , and the index  $i$  runs over all fermions,  $i = \{(t, b, \tau, \nu_\tau), (c, s, \dots), \dots\}$ . The scalar-coupling matrix  $G_s$  is taken diagonal and the dimensionful couplings are adjusted to produce the known fermion masses dynamically; in practice only the top acquires an appreciable mass. The model admits bound states corresponding to the Higgs as well as the gauge bosons of the standard electroweak theory, and is essentially equivalent to the Standard Model below some high mass scale  $\mu$ . It is the vector terms in Eq.1 which ensure the existence of the Higgs,  $Z$ , and  $W$  as composites with masses of the order of  $m_t$ , thus naturally explaining why the Standard Model bosons and the top appear to have about the same mass.

In re-examining the predictions for the top and Higgs we do not presume to seek precise values for their masses, but rather attempt to determine the latitude in masses present in the modeling. Such a study is especially timely in light of the search for the top being carried out at FNAL [9]. The apparent paucity of top events in the latest data suggests a high mass for the top, certainly it now seems  $m_t$  is greater than 120 GeV and possibly considerably higher. Present analyses of LEP data [10] with respect to EW corrections, suggest  $m_t = 166 \pm 30$  GeV.

As usual in NJL the necessary fine tuning of the scalar coupling is accomplished by solving the scalar gap equation, whence diagonalisation of the scalar action yields the Higgs mass formula:

$$m_H(\mu) = 2m_t(\mu)(1 + O(g_t^2)) \quad (2)$$

Fine tuning determines the dimensionful scalar coupling in terms of the cutoff  $\Lambda$ ,

$$G_t^2 = \frac{1}{\Lambda^2 - m_t^2 \ln\left(\frac{\Lambda^2}{m_t^2}\right)} \quad (3)$$

Bound states also exist in the vector sector defined by Eq.1 corresponding to the  $W$ ,  $Z$ , and the photon. A similar fine tuning of the vector coupling is required, but here with the added physical interpretation that the photon mass should vanish [1]. This latter

constraint leads, at lowest order in the electroweak and Yukawa couplings, to the mass relationship

$$m_W^2(\mu) = \frac{3}{8} m_t^2(\mu) \quad (4)$$

To the same order in couplings, the required diagonalisation of the neutral vector boson action results in

$$\sin^2(\theta_W) = (\sum_i Q_i^2)^{-1} = \frac{3}{8}, \quad (5)$$

with the denominator on the right hand side of Eq.(5) being summed over the charges  $Q_i$  in one generation.

The dimensionful couplings of the 4-fermion theory are replaced, after fine-tuning and wave function renormalisation, by the dimensionless couplings of the Standard Model [11,1], and the gradient expansion of the effective action is in fact an expansion in these dimensionless electroweak couplings. One has for the scalars

$$\begin{aligned} g_S &= G_S Z_S^{-\frac{1}{2}}, \\ Z_S &= \frac{1}{2} \text{Tr} \left[ G_S^2 \frac{1}{(\partial^2 + M^2)^2} \right], \end{aligned} \quad (6)$$

where the fermion–scalar coupling matrix is for the present taken diagonal:

$$(G_S)_{ij} = G_i \delta_{ij}. \quad (7)$$

Similarly, for the vector couplings one has

$$\frac{g_2}{2} = \frac{G_W}{\sqrt{Z_W}} \quad \text{and} \quad \frac{g'}{2} = \frac{G_B}{\sqrt{Z_B}} \quad (9)$$

and the usual relationship between  $g_2$  and  $g'$

$$g_2 \sin(\theta_W) = g' \cos(\theta_W). \quad (10)$$

From equations  $\{(2), (4), (5)\}$ , valid presumably at a scale  $\mu$  where the cross coupling between the EW and strong sectors is small but still well below the cutoff  $\Lambda$ , we derived values for the top and Higgs masses at a scale near  $m_W$ . The theory leading to these equations is equivalent to the electroweak sector of the Standard Model below  $\mu$ , and the framework for connecting the scales  $\mu$  and  $m_W$  is provided by the Standard Model RG. Thus  $SU(3)_c$  influences on the top and Higgs masses are included through the renormalisation group, below the matching scale  $\mu$ .

### 3. Renormalisation Group Evolution

We turn now to the calculation of smaller effects, neglected in the initial work, due to corrections in the 4-fermion theory of higher order in the electroweak couplings and to a more consistent treatment of the evolution downward to experimental mass scales. Our basic equations are: (1) the boundary condition relationships between the Higgs, top and W masses including dependence on electroweak couplings and quark masses, and (2) the RG evolution equations for the top-Higgs and Higgs-self couplings  $g_t$  and  $\lambda$ . Defining [12,13]

$$\kappa_t = \frac{g_t^2}{2\pi},$$

one has

$$\frac{d\kappa_t}{dt} = \frac{9}{4\pi}\kappa_t^2 - \frac{4}{\pi}\kappa_t\alpha_S - \frac{9}{8\pi}\kappa_t\alpha_W - \frac{17}{4\pi}\kappa_t\alpha_1, \quad (12)$$

with  $\alpha_S, \alpha_W, \alpha_1$  taken equal to  $\alpha_3, \alpha_2, \alpha_1$  respectively in reference [12,13], and  $t = \ln(\frac{q}{m})$ . We note that with these choices

$$\begin{aligned} m_t &= g_t v, \\ m_W &= \frac{g_W}{2}v, \end{aligned} \quad (13)$$

where  $v$  is the standard EW vev.

Also taking  $m_H^2 = 2\lambda v^2$  the evolution equation for the Higgs self-coupling is, to the same (one-loop) order [14]:

$$\frac{d\lambda}{dt} = \frac{1}{16\pi^2} \left\{ 12\lambda^2 + 6\lambda g_t^2 - 3g_t^4 - \frac{3}{2}\lambda \left( 3g_W^2 + g'^2 \right) + \frac{3}{16} \left( 2g_W^4 + \left( g_W^2 + g'^2 \right)^2 \right) \right\}. \quad (14)$$

Redefining the standard choice of couplings [12]

$$\alpha_1 = \frac{5}{3}\alpha' \quad \text{with} \quad \alpha_1 = \frac{g_1^2}{4\pi}, \quad \alpha' = \frac{g'^2}{4\pi} \quad (15)$$

and setting

$$\sigma = \frac{\lambda}{4\pi} \quad (16)$$

results in

$$\frac{d\sigma}{dt} = \frac{1}{2\pi} \left\{ 12\sigma^2 + 6\sigma\kappa_t - 3\kappa_t^2 - \frac{9}{2}\sigma \left( \alpha_W + \frac{1}{5}\alpha_1 \right) + \frac{3}{16} \left( 2\alpha_W^2 + \left( \alpha_W + \frac{3}{5}\alpha_1 \right)^2 \right) \right\}. \quad (17)$$

Equations (2) and (4) impose boundary conditions on equations (12) and (17) at the scale  $\mu$ . These are to lowest order

$$m_t^2 = \frac{8}{3} m_W^2$$

and

$$m_H^2 = 4m_t^2 = \frac{32}{3} m_W^2, \quad (18)$$

which can be restated to include higher orders:

$$\frac{\kappa_t}{\alpha_2}(\mu) = \frac{4}{3} + O(g_i^2) \quad \text{and} \quad \frac{\sigma}{\alpha_2}(\mu) = \frac{4}{3} + O(g_i^2). \quad (19)$$

Such corrections can come from two sources, higher order  $1/N$ , multi-loop, contributions to the effective action, and more trivial  $1/\ln(\Lambda)$  terms within the lowest order. The latter arise, for example, from the proper generalised form of Eq.4:

$$m_W^2 = \frac{1}{2} \frac{\sum_i m_i^2 \left[ \ln \left( \frac{\Lambda^2}{m_i^2} + 1 \right) - 1 \right]}{\sum_i \frac{r_i}{6} \left[ \ln \left( \frac{\Lambda^2}{m_i^2} + 1 \right) - \frac{11}{6} \right]}, \quad (20)$$

where the sum is over all fermions and  $r_i = \alpha^{-2} (\beta^4 (y_{Li}^2 + y_{Ri}^2) - \beta^2 \alpha^2 y_{Li} \tau_i^3 + \alpha^4 \tau_i^3 \tau_i^3)$ , while  $y_{Li}$ ,  $y_{Ri}$  and  $\tau_i^3$  are the fermion hypercharges and isospins. Equation (4) is obtained from (20) by keeping only the top mass and ignoring terms of order  $(\ln(\Lambda))^{-1}$ . These terms are of higher order in the electro weak couplings; for example the Higgs-top Yukawa coupling is, from (6), proportional to  $(\ln(\Lambda))^{-1}$ .

We note parenthetically that the basic  $SU(5)$  symmetry evident in Eqs(4,5) results from the **5 + 10** generational structure  $(u, d, e_{L,R}, \nu_R)$  built into the present model, and follows from (20) in the limit of large  $\Lambda$ . We also note that the  $\frac{3}{8}$  appearing in the lowest order (Eq(4)) for  $m_W^2$  is more properly written

$$\frac{m_W^2}{m_t^2} = \frac{3}{8} \frac{n_g}{n_c}, \quad (21)$$

and so is not simply  $\sin^2(\theta_W)$  but instead depends on the number of colours as well as the number of massive fermion generations. We find that the several percent change implied in Eq(20) relative to Eq(4) produces a considerably smaller change in  $m_t$ , less than one percent. Thus, to the accuracy meaningful here, we can perhaps ignore these corrections as well as other higher order  $1/N$  effects arising from discarded, incoherent, summations over fermions.

#### 4. Solution of the RG Equations.

It is possible to obtain an explicit solution to Eq(12), and a perturbative solution for Eq(17). For the top evolution one has, making a simple transformation of Eq(12)

$$\frac{d}{dt} \frac{1}{\kappa_t} = -\frac{9}{4\pi} + \frac{1}{\kappa_t} \left( \frac{4}{\pi} \alpha_S + \frac{9}{8\pi} \alpha_W + \frac{17}{40\pi} \alpha_1 \right), \quad (22)$$

with the one parameter family of solutions

$$\begin{aligned} \frac{1}{\kappa_t} = & \left[ \frac{(1 + \alpha_{S0} b_S t)^{8/7} (1 + \alpha_{W0} b_W t)^{27/38}}{(1 - \alpha_{10} b_1 t)^{17/82}} \right] \\ & \times \left[ D - \frac{9}{4\pi} \int_0^t dt' \frac{(1 - \alpha_{10} b_1 t')^{17/82}}{(1 + \alpha_{S0} b_S t')^{8/7} (1 + \alpha_{W0} b_W t')^{27/38}} \right] \end{aligned} \quad (23)$$

Here  $\alpha_{S0}$ ,  $\alpha_{W0}$ , and  $\alpha_{10}$  are the couplings at  $t = \ln \frac{m_W}{m_W} = 0$ , and the constants  $b_S = \frac{7}{2\pi}$ ,  $b_W = \frac{19}{12\pi}$  and  $b_1 = \frac{41}{20\pi}$  determine the evolution of the  $SU(3)$ ,  $SU(2)$ , and  $U(1)$  couplings respectively. The constant  $D$  in Eq(23) is given by

$$D = \frac{1}{\kappa_t(0)}, \quad (24)$$

and directly yields the running top mass at the scale  $m_W$  from

$$m_t^2(m_W) = \frac{2\kappa_t(0)}{\alpha_W(0)} m_W^2(m_W). \quad (25)$$

To self-consistently determine the physical top mass as a pole in the top quark propagator, one must run  $m_t(m_W)$  back up to get  $m_t(m_t)$ .

The cross coupling in Eq(17) complicates its solution. The pure scalar self-coupling result

$$\sigma_0(t) = \frac{\sigma_0(0)}{1 - \frac{6}{\pi} \sigma_0(0) t}, \quad (26)$$

may be improved perturbatively

$$\sigma(t) = \sigma_0(t) + \sigma_1(t). \quad (27)$$

Linearising in the small correction  $\sigma_1(t)$  produces

$$\sigma_1(t) = e^{-\nu(t)} \int_{t_\mu}^t dt' g(t') e^{\nu(t')}, \quad (28)$$

with

$$v(t) = - \int_{t_\mu}^t dt' f(t'), \quad (29a)$$

$$f(t) = \frac{12}{\pi} \sigma_0(t) + \frac{3\kappa_t(t)}{\pi} - \frac{9}{4\pi} \left[ \alpha_2(t) + \frac{1}{5} \alpha_1(t) \right], \quad (29b)$$

and

$$g(t) = \frac{3}{\pi} \sigma_0(t) \kappa_t(t) - \frac{3\kappa_t^2}{2\pi} + \frac{3}{32\pi} \left[ 2\alpha_2^2(t) + \left( \alpha_2(t) + \frac{3}{5} \alpha_1(t) \right)^2 \right]. \quad (29c)$$

Boundary conditions are introduced at  $t_\mu = \ln \frac{\mu}{m_W}$  through

$$\sigma_1(t_\mu) = 0, \quad \sigma_0(t_\mu) = \kappa_t(t_\mu) = \frac{4}{3} \alpha_2(t_\mu) + O(\alpha_i^2). \quad (30)$$

Since  $\sigma_1(t)$  is small over the range  $m_H$  to  $\mu$  (see Fig.1) there is no need to include higher orders.

Results from numerical integration of equations (23) and (28,29) are displayed in Table 1, and Figs 1-4. We have varied the inputs to these calculations, the strong and electroweak couplings  $\alpha_{i0}$ ,  $i = 1, W, S$  over a reasonable range, somewhat wider than the flexibility allowed by present experiments. The W mass is fixed at 80.1 GeV. There are no free parameters in the theory, the couplings and  $m_W$  being determined from experiment. A possible exception is the cutoff  $\Lambda$ , which is surely well above  $\mu$  and has essentially no effect on  $m_t$  and  $m_H$ . Any dependence other than logarithmic on  $\Lambda$  has been eliminated by fine tuning, while residual  $\ln(\Lambda)$  presence is transmuted into dependence on the dimensionless couplings.

The effect of imposing boundary conditions sharply at a scale  $\mu$  remains to be examined. As we noted above,  $\mu$  is that point, when one is evolving downward in mass, at which the  $g_i$  become interdependent. For example, the top quark evolution is strongly influenced by  $SU(3)_c$  from  $\mu \sim 10^{14}$  downward, and the running of  $\alpha_W$  is also significant. Varying  $\mu$  over four orders of magnitude from  $\mu = 10^{10}$  GeV to  $\mu = 10^{14}$  GeV has practically no effect on  $m_t$ , and only a small effect on  $m_H$ . This remarkable result is demonstrated in Fig.(2) for central choices of the couplings, and lends credence to our use of a sharp boundary condition.

The one physical parameter sensitive to  $\mu$  is the weak mixing angle  $\theta_W$ . We indicated [1] that, for one loop evolution,  $\sin^2(\theta_W)$  achieves its experimental value  $\sim 0.23$  (at  $m_W$ ) for  $\mu \sim 10^{13}$  GeV. Unlike GUTS, the present theory need not have a single scale at which the gauge couplings are equal. The unification present in this model simply implies that

the Standard Model should evolve smoothly into the effective 4-fermion theory where the couplings become weak. Table 1 displays the value of the couplings at scale  $\mu$ ; the  $\alpha_i$  are the experimental values determined at  $m_W$  evolved upward to  $\mu$  at 1-loop and  $\kappa_t(\mu)$  is obtained from the boundary condition  $\frac{\kappa_t}{\alpha_2} = \frac{4}{3}$ . It is clear that the couplings are indeed all small at  $\mu$ , again justifying the placing of the boundary conditions there.

Figures (3) and (4) show the variations of  $m_t$  and  $m_H$  with the strong and electroweak couplings, respectively. The strong coupling is less well known. Using as central values  $\alpha_{S0} = 0.107$ ,  $\alpha_{W0} = 0.0344$ , and  $\alpha_{10} = 0.0169$  [10,16], we get  $m_t \simeq 175$  GeV and  $m_H \simeq 125$  GeV. Included in the 175 GeV is a 6 GeV reduction from evolving the top self-consistently to its proper mass at  $q = m_t$ ; for the Higgs this effect is much smaller. Further small contributions to Eq(19), from non-leading log terms in defining the top pole and from running the W mass, more or less cancel. It is clear from the figures that  $m_H$  is somewhat more sensitive to all these changes, and so the remaining uncertainty in the mass 125 GeV is larger. This uncertainty nevertheless may be usefully bounded by noting [1] that a rather large arbitrary variation in the boundary condition ratio  $m_H/m_t$  from 2 to  $\sqrt{8}$  produces  $\leq 15$  GeV change in  $m_H$ . One must also keep in mind that the top is confined and its mass therefore subject to some ambiguity in definition.

#### 4. Conclusions.

In summary, one gets remarkably stable predictions for the top and Higgs masses and in a parameter free fashion. The only inputs were the experimentally known couplings and the W-mass. A characteristic prediction of this type of theory is  $m_h < m_t$ , so that the Higgs, which is practically a  $t\bar{t}$  condensate, is deeply bound.

In view of the present dearth of events from the FNAL experiments with DØ and CDF, the above prediction for the top (near 175 GeV) may not be wholly wild. In light of the recent unfortunate developments at the SSC, the somewhat low prediction for the Higgs mass, near 125 GeV, may take considerably longer to test.

Finally, there is the question of the number of generations. In Ref(1), we indicated that a fourth generation, with massive quarks  $m_{t'} \sim m_{b'} \sim m_t$ , implies a top mass near 115 GeV. Such a constraint arises from the sum rule (Eq(19)) for  $m_W^2$ . Present data at FNAL appear to rule out this possibility.

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## References

1. D.E. Kahana and S.H. Kahana, *Phys. Rev.* **D43**, 2361 (1991)
2. Y. Nambu and G. Jona-Lasinio, *Phys. Rev.* **122**, 345 (1961)
3. J.D. Bjorken, *Ann. Phys. (NY)* **24**, 174 (1963)
4. W. Bardeen, C.T. Hill, and M. Lindner, *Phys. Rev.* **D41**, 1647 (1990)
5. H. Terazawa, Y. Chikashige, and K. Akama, *Phys. Rev.* **D15**, 480 (1977)
6. T. Eguchi, *Phys. Rev.* **D14**, 2755 (1976)
7. M. Bando, T. Kugo, and K. Yanawaki, *Phys. Rep.* **164**, 210 (1988)  
M. Suzuki, *Phys. Rev.* **D37**, 210 (1988)
8. V.A. Miranski, M. Tanabashi, and K. Yamawaki, *Mod. Phys. Lett.* **A4**, 1043 (1989)
9. Avi Yagil, *et al.*, CDF Collaboration, “Proceedings of the 7th Meeting of the American Physical Society, Division of Particles and Fields, 10-14 November, 1992.”, Vol. 1. Edited by Carl H. Albright, Peter H. Kasper, Rajendran Raja and John Yoh; World Scientific (1993), (and other contributions therein).  
Ronald J. Mahas, *et al.*, DØ Collaboration, “Proceedings of the 7th Meeting of the American Physical Society, Division of Particles and Fields, 10-14 November, 1992.”, Vol. 1. Edited by Carl H. Albright, Peter H. Kasper, Rajendran Raja and John Yoh; World Scientific (1993), (and other contributions therein).  
DØ Collaboration, in the “Proceedings of the Lepton Photon Conference, August 4-7, 1993, Cornell University, Ithaca, N.Y.
10. Samuel Ting, Summary of LEP Results, “Proceedings of the 7th Meeting of the American Physical Society, Division of Particles and Fields, 10-14 November, 1992.”, Vol. 1. Edited by Carl H. Albright, Peter H. Kasper, Rajendran Raja and John Yoh; World Scientific (1993), (and other contributions therein).  
LEP Summary, in the “Proceedings of the Lepton Photon Conference, August 4-7, 1993, Cornell University, Ithaca, N.Y.
11. G. S. Guralnik, K. Tamvakis, *Nucl. Phys.* **B148**, 283 (1979)
12. W. Marciano, *Phys. Rev. Lett.* **62**, 2793 (1989)
13. W. Marciano, *Phys. Rev.* **D41**, 219 (1990)  
W. Marciano and A. Sirlin, *Phys. Rev.* **D22**, 2695 (1980)
14. John F. Gunion, Howard E. Haber, Gordon Kane and Sally Dawson, “The Higgs Hunter’s Guide”, Addison-Wesley, New York (1990)
15. H. Georgi and S. Glashow, *Phys. Rev. Lett.* **32**, 438 (1974)

H. Georgi, H. Quinn and S. Weinberg, *ibid.* **33**, 451 (1974)

16. U. Amaldi *et al.*, *Phys. Rev.* **D36**, 1385 (1984)  
and LEP in DPF92

17. S. Fanchiotti and A. Sirlin, *Phys. Rev.* **D41**, 319 (1990)

## Figure Captions

**Fig. 1** Evolution of the reduced Higgs Self Coupling  $\sigma = \sigma_0 + \sigma_1$  over the range from  $m_W$  to  $\mu = 10^{14}$ . The perturbation  $\sigma_1$  remains small.

**Fig. 2** Variation of the top and Higgs masses with the matching scale  $\mu$  over a range from  $10^{10}$  to  $10^{14}$  GeV. The scale  $\mu = 7.5 \times 10^{12}$ , for which  $\sin^2(\theta(\mu)) = \frac{3}{8}$ , is defined as a ‘central value’.

**Fig. 3** Variation of  $m_t$  and  $m_H$  with the strong coupling;  $\alpha_S = 0.107$  is considered the central value.

**Fig. 4** Variation of  $m_t$  and  $m_H$  with the weak coupling;  $\alpha_W = 0.0344$  is the central value.

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## Top and Higgs Masses vs $\mu$

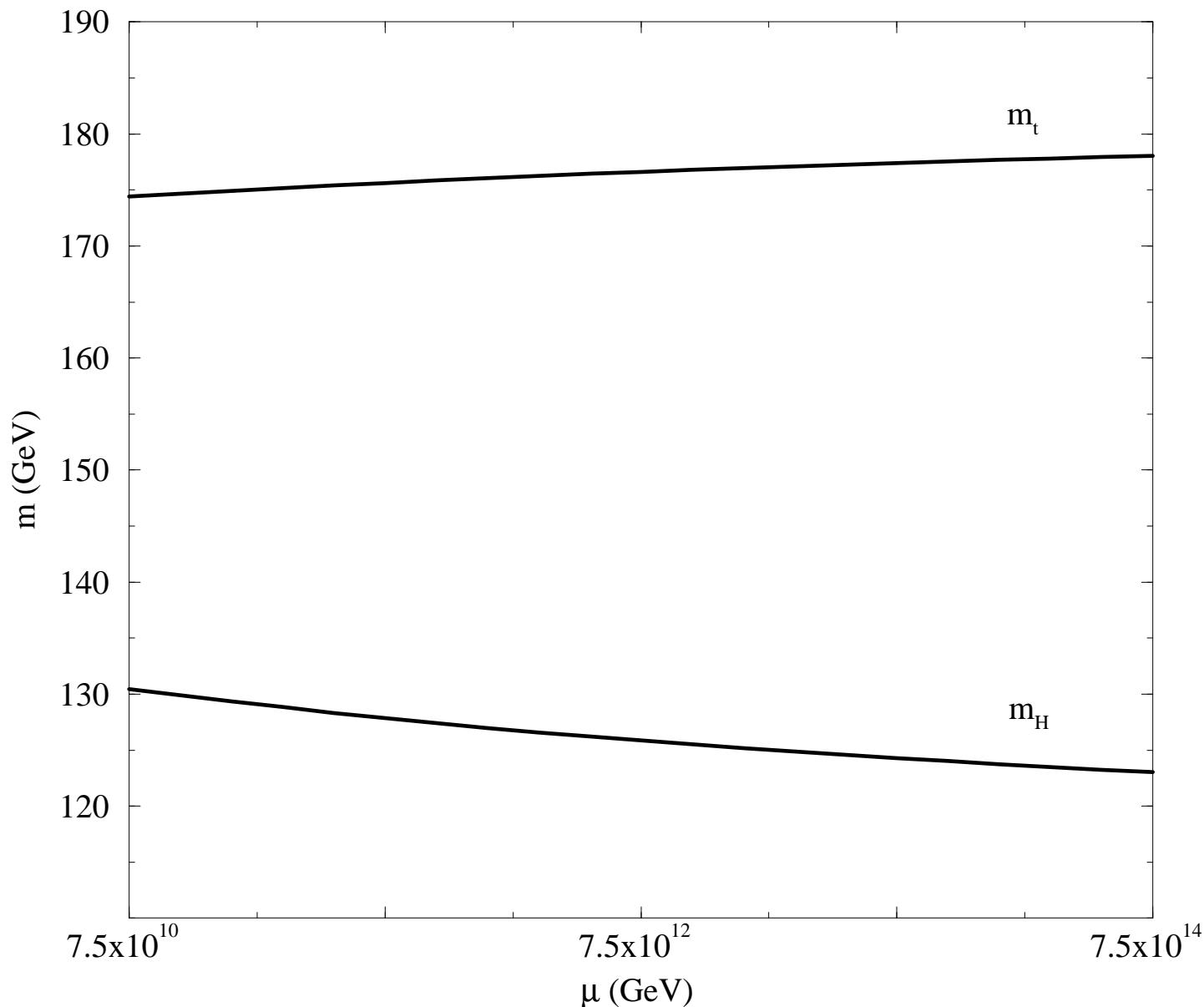


Fig. 2

# Evolution of Higgs Self Coupling

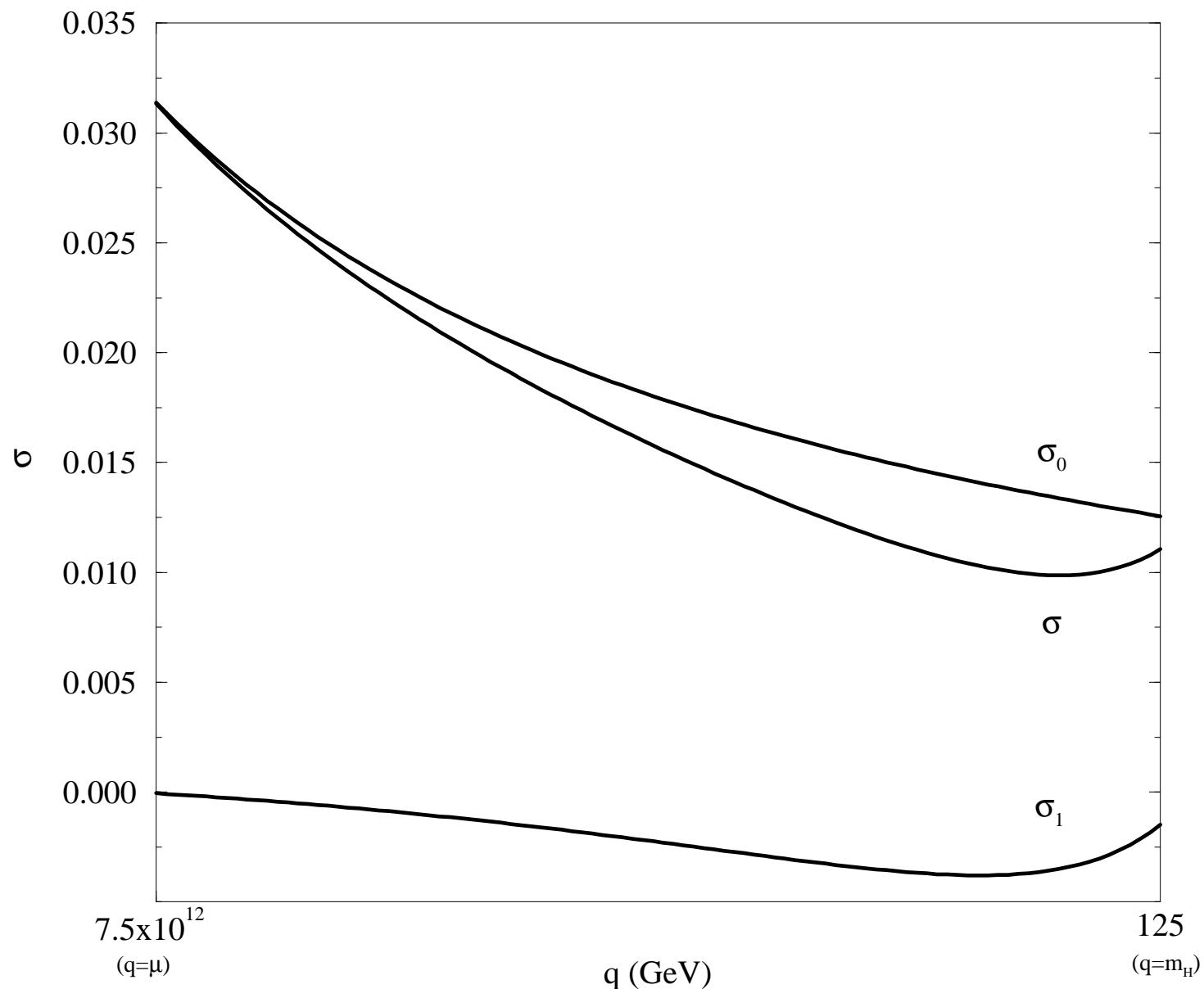


Fig. 1

# Top and Higgs Masses vs $\alpha_s(m_W)$

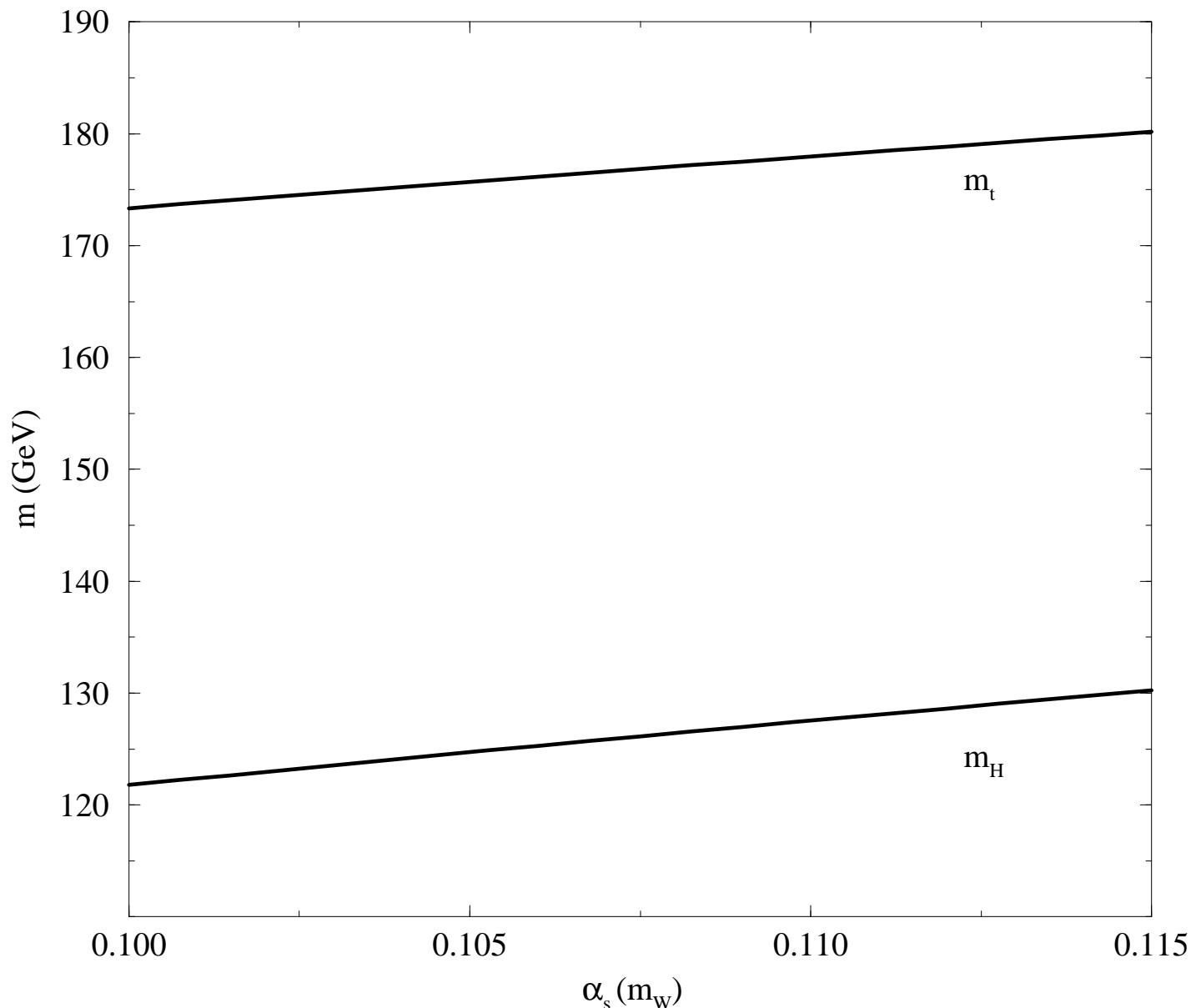


Fig. 3

**Table 1**

The  $SU(3)$ ,  $SU(2)$ , and  $U(1)$  couplings, as well as the Higgs-top, and the Higgs-self coupling are shown at both the scales  $m_W = 80.1$  GeV and  $\mu = 7.5 \times 10^{12}$  GeV. At the upper scale, all these couplings are comparable, and may be considered small.

$q$	$\alpha_S$	$\alpha_W$	$\alpha_1$	$\kappa_t$	$\sigma$
$m_W$	.107	.0344	.0169	.0880	.0111
$\mu$	.0267	.0239	.0234	.0319	.0319

## Top and Higgs Masses vs $\alpha_w(m_w)$

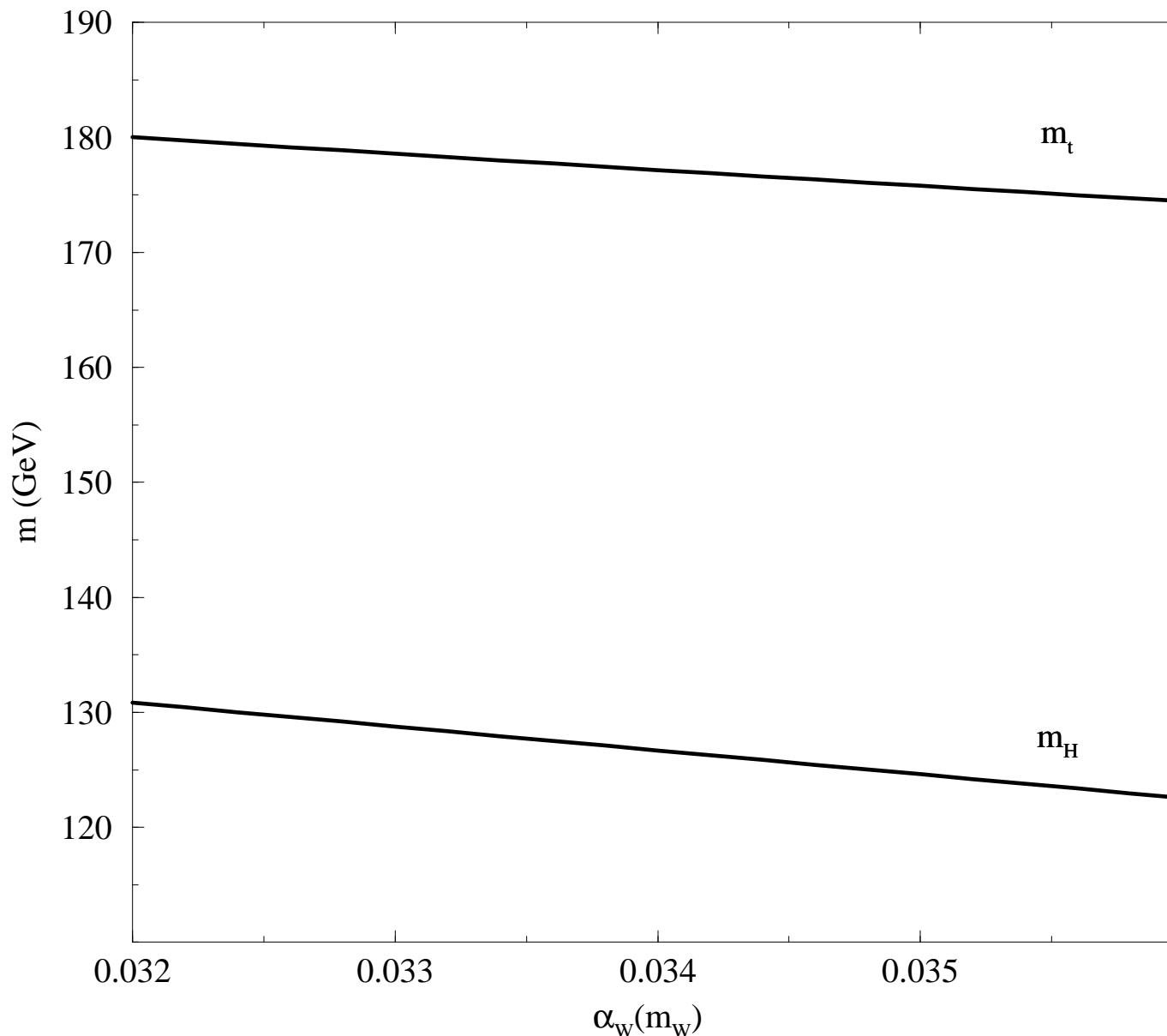


Fig. 4